Exam Introduction to Logic (CS)

Thursday 9 November 2017, 14 - 17 h.

- Write your student number on the first page. **Do not write your name** so as to enable anonymous grading. Write with a *blue or black pen* (so no pencil, no red pen). Only hand in your definite answers. You can take the exam questions and any drafts home.
- With the regular exercises, you can earn 100 points. With the bonus exercise, you can earn additionally 10 points. The exam grade is the number of points you earned divided by 10, with a maximum of 10. The final grade F is computed as

$$F = 0.08 \cdot H_1 + 0.16 \cdot H_2 + 0.16 \cdot M + 0.60 \cdot E.$$

Here H_1 is the grade for homework assignment 1, H_2 is the grade for homework assignment 2, M is the midterm exam grade, and E is the grade for this exam.

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1. (10 points) Translate the following sentences to *propositional logic*. Atomic sentences are represented by uppercase letters. Do not forget to provide the translation key.

   (a) Unless he forgets the date, Philip will attend the lecture if he is in Groningen.

   (b) Only if Anna finds a spare moment, she will meet Karim, although neither she nor Karim is in good health.

2. (15 points) Translate the following sentences to *first-order logic*. The domain of discourse is the set of students. Use the following translation key:

   e: Eva
   m(x): the mentor of x
   K(x,y): x knows y

   (a) There are exactly two students who do not know their mentor.

   (b) No student knows the mentors of all students.

   (c) Eva is the mentor of a student who knows the mentor of the mentor of Eva.

3. (10 points) Answer the following questions using truth tables. Write down the complete truth tables and motivate your answers. Order the rows in the truth tables as follows.

In (a):

A	B	C	...
T	T	T	...
T	T	F	...
T	F	T	...
T	F	F	...
F	T	T	...
F	T	F	...
F	F	T	...
F	F	F	...

and in (b):

A	B	...
T	T	...
T	F	...
F	T	...
F	F	...

   (a) Are the following two sentences tautologically equivalent?

      i. $\neg((A \leftrightarrow B) \vee C)$

      ii. $A \wedge \neg(B \vee C)$

   (b) Is $(A \leftrightarrow B) \leftrightarrow ((A \vee B) \rightarrow (A \wedge B))$ a tautology?

4. (32 points) Give formal proofs of the following inferences. Do not forget the justifications. You may only use the Introduction and Elimination rules and the Reiteration rule.

   (a) $\vdash (A \to B) \vee (A \to \neg C)$
   $\quad\quad (A \wedge \neg B) \to \neg C$

   (b) $\vdash \exists x(A(x) \wedge \neg B)$ (variable x does not occur in B)
   $\quad\quad \neg(\exists x A(x) \to B)$

   (c) $\vdash \forall x(A(x) \to \neg(x = a))$
   $\quad\quad A(b) \to \exists y \exists z\, \neg(y = z)$

   (d) $\vdash \neg\exists x\, \neg A(x)$
   $\quad\quad \forall x A(x)$

5. (18 points) Let a model $\mathfrak{M}$ with domain $D = \mathfrak{M}(\forall) = \{1, 2, 3\}$ be given such that

$$\mathfrak{M}(a) = 1 \quad \mathfrak{M}(b) = 2$$
$$\mathfrak{M}(P) = \{1, 3\}$$
$$\mathfrak{M}(R) = \{\langle 1, 2\rangle, \langle 2, 3\rangle, \langle 3, 1\rangle\}$$

   Let h be an assignment such that $h(x) = 2$, $h(y) = 3$, and $h(z) = 2$. Evaluate the following statements. Follow the definition of satisfaction (truth definition) step by step.

   (a) $\mathfrak{M} \models \neg\forall x P(x) \to (R(a, y) \vee P(b))[h]$

   (b) $\mathfrak{M} \models R(a, b) \wedge \exists y(P(y) \wedge R(a, y))[h]$

   (c) $\mathfrak{M} \models \forall x \exists y R(x, y)[h]$

6. (15 points)

   (a) Provide a disjunctive normal form (DNF) of the sentence $\neg((C \vee \neg D) \wedge ((\neg A \wedge B) \vee C))$. Show all intermediate steps.

   (b) Provide a Skolem normal form of the sentence $\forall x \exists z Q(x, z) \to \exists x \forall y R(x, y)$. Show all intermediate steps.

   (c) Use the Horn algorithm to check the satisfiability of the Horn sentence below.

$$A \wedge (C \to D) \wedge ((E \wedge A) \to C) \wedge ((G \wedge C \wedge D) \to \bot) \wedge (B \to C) \wedge ((A \wedge B) \to G) \wedge (A \to E)$$

7. (Bonus exercise: 10 points)
   Give a formal proof for the inference

   $\vdash P \to \exists x Q(x)$
   $\quad\quad \exists x(P \to Q(x))$

   Variable x does not occur in P. Do not forget to provide justifications. You may only use the Introduction and Elimination rules and the Reiteration rule.